مباراة الدخول للعام الجامعي 2019-2020

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مسابقة في (Data Science)

Solve the following three exercises:

Exercise 1. [30 points]

The table below shows a company's turnover over some years, in thousands of euros.

| Year | 1984 | 1985 | 1986 | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 |
|--------------------------------------|------|------|------|------|------|------|------|------|------|------|
| Rank of the Year x_i | I | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Turnover in thousands of euros y_i | 167 | 164 | 159 | 156 | 149 | 148 | 144 | 134 | 131 | 129 |

- 1. Calculate the linear correlation coefficient and interpret the answer.
- 2. Write an equation of the linear regression line $D_{y/x}$ of y on x.
- 3. Under this model, what is the turnover of this company in 2018?
- 4. In fact, the turnover in 2018 is 20 thousand of euros. Calculate the percentage of the error made with the previous estimate.
- 5. In which year the turnover reaches zero?

Exercise 2. [30 points]

A two-stage game is offered in a local casino. It consists of the player throwing a die in a first step, then in a second step he draws a token in an urn chosen according to the result of the die. The urn U1 is chosen when the die gives 1 or 2. The urn U2 is chosen when the die gives 3 or 4 and the urn U3 for 5 and 6.

The urns contain the following tokens:

- Urn U1: two red tokens, two blue tokens.
- Urn U2: two blue tokens, four green tokens.
- Urn U3: a green token, a red token.
- 1. What is the probability of obtaining a blue token given that it comes from the urn U1?
- 2. What is the probability of getting a blue token and that the token comes from the urn U1?
- 3. What is the probability that the player will get a red token?
- 4. The player gets a green token. What is the probability that this token comes from the urn U2?

Exercise 3. [40 points]

Consider the function f defined over $[0, +\infty[$ by $f(x) = (x-2)e^{-x} + 1$ and its representative curve (C) in an orthonormal system (0, i, j).

- Calculate lim _{x→+∞} f(x). Deduce an asymptote to (C).
 Determine f'(x).
- 3. Show that f'(x) can be written in the form $f'(x) = (3 x)e^{-x}$.
- 4. Determine the table of variation of f.
- 5. Sketch (C).
- 6. (a) Show that the function F defined over $[0; +\infty[$, by $F(x) = (-x+1)e^{-x} + x$ is an antiderivative of f.
 - (b) Deduce the area of the region bounded by (C), the x-axis and the lines with equations x = 0 and x = 1.