

- N.B: - Calculators are allowed.  
- No credit is awarded for an answer with no justification.

### Part A

In this part, for each exercise, indicate whether each statement is true or false and justify your answer.

#### Exercise 1 (3.5 pts)

We choose a real number  $x$  and add 10 to it; then we multiply the result by 3. We deduct 10 from the last result and we divide by 2 the final result. We obtain the number  $y$ .

- A) If  $x = 0$ , then  $y = 10$ .
- B) If  $x$  is an integer, then  $y$  is also an integer.
- C)  $y = \frac{3}{2}x + 10$
- D) It is impossible to obtain a value of 0 for  $y$ .
- E) There is one and only one value of  $x$ , such that  $x$  is equal to  $y$ .

#### Exercise 2 (2.5 pts)

$H$  is the set of real numbers strictly included between 1,234 and 1,2999.....

- A) 1.2345 belongs to  $H$ .
- B) 1.29 belongs to  $H$ .
- C)  $1 + \frac{1}{3}$  belongs to  $H$ .
- D)  $1 + \frac{1}{4}$  belongs to  $H$ .
- E)  $2 - \frac{1}{2}$  belongs to  $H$ .

#### Exercise 3 (5 pts)

During elections, there are 3960 voters who voted for one of the candidates A, B, C. The results are represented by a pie chart. The central angles of the circular sectors are  $90^\circ$  for the votes obtained by candidate A, and  $137^\circ$  for the votes obtained by candidate B

- A) C won.
- B) A obtained one fourth of the votes.
- C) There is less than 1% difference between the votes for B and C
- D) A obtained 25 votes.

E) B had 32 votes more than C.

**Exercise 4 (3 pts)**

- A) The sum of two even numbers is even.
- B) The sum of two odd numbers is odd.
- C) The product of two even numbers is even.
- D) The product of two odd numbers is odd.
- E) If  $n$  is an integer, then  $n^2+n$  is odd.

**Exercise 5 (3 pts)**

Given that  $x < 0$

- A)  $|-x| = x$
- B)  $\sqrt{x^2} = x$
- C)  $\sqrt{(x-3)^2} = x-3$
- D)  $|3-x| = 3-x$
- E)  $|x-2| = x+2$

**Part B**

In this part, for each exercise, justify your answer.

**Exercise 1 (2 pts)**

We pick at random a card from a deck of 52 cards. Find the probability of drawing neither an ace nor a club?

**Exercise 2 (5 pts)**

Consider the numbers:  $x = \frac{5}{6}$  and  $y = \frac{4}{9}$

- A) Compare  $x$  and  $y$
- B) Calculate the sum of the inverses of  $x$  and of  $y$ .
- C) Calculate the inverse of the sum of  $x$  and  $y$ .
- D) Calculate the product of the opposites of  $x$  and of  $y$ .
- E) Find the missing factor  $x = \dots \times y$

**Exercise 3 (5 pts)**

The quarterly average of a student is calculated from 5 homeworks, each graded out of 20. Alan has a average of 14.5 on 4 homeworks.

- A) Could his grades be 13, 14, 15 and 16 ?
- B) Could this average be obtained with one of the grades being 0?
- C) If Alan gets a 17 on his fifth homework, what would his quarterly average be?

- D) Alan claims: «Whatever my score would be on the fifth homework, my average would at least be equal to 12 ». Is he right?

**Exercise 4 (2.5 pts)**

Calculate  $e^{\frac{1}{2}\ln 4} + e^{-\ln \frac{1}{2}}$

**Exercise 5 (2.5 pts)**

Calculate the derivative of  $f(x) = \ln(x^2 + 1) - x$ , for any real  $x$ .

**Exercise 6 (5 pts)**

On a particular day, 2650 people visited the Louvre museum. The entrance fee is 10€ per adult and half price per child. The net income for that day was 20500€. How many adults and children visited the Louvre on that day?

**Exercise 7 (7 pts)**

ABCD is a rhombus with triangle ABD being equilateral. I is the midpoint of [AB], J is the midpoint of [BC], L is the midpoint of [DA], K is the midpoint of [DC].

- A) Show that the triangle LBK is equilateral.  
B) What is the nature of the quadrilateral IJKL ? Justify.

**Exercise 8 (4 pts)**

Alice wants to find the height ST of a tree. She stands at 25 meters from the foot of the tree on a level ground, her eye O being 1.60 m above ground. Her brother sticks a 2.5 m tall pole in the ground at 3.5 meters away from Alice, such that her eye O, the tip A of the pole, and the top S of the tree are aligned. She draws the figure below where (ST) and (AB) are parallel. Calculate the height of the tree.

