الجامعة اللبنانية كلية العلوم الاقتصادية وادارة الاعمال	مباراة دخول للعام : ٢٠١٧-٢٠١٨ فرع الإقتصاد	السبت في ١٥ تموز ٢٠١٧
عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

The table below represents the evolution of number of persons aged 64 and above in a certain city.

	1980	1985	1990	1995	2000	2005	2010
x_i : rank of year	0	5	10	15	20	25	30
y_i : number of persons in thousands on the first of January of each year	210	230	290	360	420	490	560

We choose an orthogonal system has as graphical units: 1 cm for 5 years on the x-axis and 1 cm for 100 000 persons on the y-axis.

1) a- Find the coordinates of the average point G (center of gravity) of the distribution $(x_i; y_i)$.
   b- Determine an equation of the regression line ($D_{y/x}$) of this distribution. (Round the values of a and b to the nearest hundredth).
   c- Represent the scatter plot of points $(x_i; y_i)$, the point G and draw the line ($D_{y/x}$).
2) Calculate the coefficient of correlation of this series. Interpret the result.
3) Assume that the preceding model remains valid till the year 2040.
   a- Estimate the number of persons of this city that are aged 64 and above in the year 2025.
   b- Determine the year after which the number of persons, in this city, aged 64 and above exceeds 780 000 for the first time.

II- (4 points)

A restaurant proposes to its clients the following formula: a daily dish and the choice of one dessert (apple pie or ice-cream) with or without coffee.

A client might choose an apple pie, an ice-cream, or none of them. The client cannot choose both desserts. We notice that:

50% of clients choose ice-cream.
30% of clients choose apple pie.
20% of clients do not choose any dessert.
Out of the clients choosing ice-cream, 80% choose coffee.
Out of the clients choosing apple pie, 60% choose coffee.
Out of the clients not choosing any dessert, 90% choose coffee.

One client from the restaurant is randomly chosen and interviewed. Consider the following events:

G: «The client chooses ice-cream» T: «The client chooses apple pie»
N: «The client does not choose any dessert» C: «The client chooses coffee»

1) a- Calculate the probabilities $P(G \cap C)$ and $P(T \cap C)$.
   b- Verify that P(C) = 0.76 .
2) a- Verify that $P(\overline{C} \cap \overline{G}) = 0.14$.
   b- Knowing that the client does not choose coffee, calculate the probability that he does not choose ice-cream.
3) The price of an ice-cream is 4 000 LL, of an apple pie is 4 000 LL, and of coffee is 3 000 LL. Each client chooses one daily dish only of fixed price of 18 000 LL.
   Let X be the random variable that is equal to the sum, in LL, paid by a client in this restaurant.
   a- Verify that the four possible values of X are: 18 000 , 21 000 , 22 000 and 25 000.
   b- Prove that $P(X = 22\,000) = 0.22$ and calculate $P(X = 25\,000)$.

III- (4 points)

On the first of January 2015, Nadim deposits in a bank a sum of x LL with an interest annual rate of 6% compounded yearly. In addition, on the first of January of each coming year, and after the capitalization of the interest, Nadim adds the amount of 1 800 000 LL to the account. Let $U_0 = x$ and, for every natural number n, let U_n be the amount in this account on the first of January of year (2015 + n).

1) For every natural number n, justify that $U_{n+1} = 1.06\,U_n + 1\,800\,000$.
2) For every natural number n, let $V_n = U_n + 30\,000\,000$.
   a- Verify that the sequence (V_n) is a geometric sequence whose common ratio should be determined. Express, in terms of x, the first term of (V_n).
   b- Express U_n in terms of x and n.
3) Calculate the value of x so that the amount in the account will be 197 245 852.8 LL on the first of January 2019.

IV- (8 points)

Part A

Let f be the function defined over $]0; +\infty[$ as $f(x) = x - 1 - 2\ln x$. Denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Determine $\lim\limits_{\substack{x \to 0 \\ x > 0}} f(x)$, then deduce an asymptote to (C).
   b- Determine $\lim\limits_{x \to +\infty} f(x)$.
2) Calculate $f'(x)$ and set up the table of variations of function f.
3) Prove that the equation f(x) = 0 has exactly two roots 1 and α. Verify that $3.5 < \alpha < 3.52$.
4) Calculate f(5) and f(7), then draw (C).

Part B

A factory produces a liquid detergent. The daily production is included between 25 and 500 liters. Assume that all production is sold.

In what follows, the costs and the revenues are expressed in hundreds of thousands of LL.

Denote by x the daily production expressed in hundreds of liters and we define the total cost C of production as $C(x) = x^2 - 2x\ln x$ where $x \in [0.25; 5]$.

Suppose that the sale price of one liter is **1000 LL.**

1) Calculate, in LL, the cost of production of 100 liters of this detergent.
2) a- Express, in terms of x, the revenue R(x).
   b- Justify that $P(x) = -x\,f(x)$ is the profit realized by the factory for the sale of x hundreds liters.
3) a- Does the factory make a gain if the daily production exceeds 360 liters? Justify.
   b- Determine the production in liters where the profit moves from negative to positive. Interpret the obtained value economically.