الجامعة اللبنانية
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مباراة الدخول الى معهد الفنون الجميلة للعام ٢٠١٧-٢٠١٨

قسم الهندسة المعمارية — مسابقة في الرياضيات (انكليزي) — المدّة: ساعتان

I-(9 points). The space is referred to a direct orthonormal system (O; $\vec{u}$, $\vec{v}$), consider the point M with affix $z = x + iy$ where x and y are two real numbers

1. a) Prove that, if Re$[z(1+i)]$ + $z\bar{z}$ = 0 then, M belongs to a circle of center and radius to be determined

b) Prove that, if Im$[(2-i)z]$ = 1 then, M belongs to a straight line of equation to be determined

c) Prove that there is no complex number such that $|z| - z$ = i

2. Consider the complex number $z' = \frac{(3+4i)z+5\bar{z}}{6}$

a) Prove that, if $z' = z$ then, M belongs to a straight line (D) of equation $y = x/2$

b) Assuming that z is pure imaginary, find tan(arg z').

II-(5 points). Given the circle (C) of equation $x^2 + y^2 - 4y = 0$

1. Find the coordinates of its center, its radius R, and the coordinates of its points of intersection A and B with the y- axis.

a) Consider the straight line (d_m) of equation $mx + y - 1 = 0$ where m is a real parameter Show that $\forall\, m$, the straight line (d_m) passes through a fixed point of coordinates to be determined. Deduce that $\forall\, m$, (d_m) cuts (C).

b) Prove that the abscissas of the points of intersection of circle (C) with line(d_m) satisfy the equation

$$(1 + m^2)\, x^2 + 2mx - 3 = 0$$

Again, deduce that, $\forall\, m \in R$, (d_m) cuts (C) at 2 points

III-(12 points). A function f is defined on IR by f(x) = $\frac{1}{5}(ax+b)e^{-x}$ where a and b are real numbers. The representative curve (γ) of f passes through point A(0; 2) and has a maximum value at the point of abscissa $x = -1$.

1. Find the values of a and b
2. Solve the equation $f(x) = 0$ where f(x) = $(x+2)e^{-x}$.
3. Find $\lim\limits_{x\to -\infty} f(x)$, $\lim\limits_{x\to +\infty} f(x)$, and deduce the asymptote to (γ)
4. Establish the table of variation of f on IR and plot its graph (γ) in an orthonormal system of coordinates. **(Unit: 2 cm)**
5. Show that F(x) = $-(x+3)e^{-x}$ is a primitive of f(x) and find the area of the region Δ bounded by the curve (γ), the line $x = 0$, the line $x = -2$, and the x-axis
6. Study, referring to curve (γ), the number and sign of the roots of the equation $m - f(x) = 0$ for $m \in [0; e]$
7. Let $g(x) = \ln[f(x)]$ where $x > -2$

   a) Find $\lim\limits_{x\to -2^+} g(x)$ and $\lim\limits_{x\to \infty} g(x)$ by using the previous results and the curve (γ).

   (Hint: if h is a continuous function at c then, $\lim\limits_{x\to c} \ln[h(x)] = \ln[\lim\limits_{x\to c} h(x)]$

   b) Find $g'(x)$ and establish the table of variation of $g(x)$ on the interval $]-2, +\infty[$

IV-(8 points). The space is referred to a direct orthonormal system $(O;\vec{i},\vec{j},\vec{k})$, consider the plane

(P_m): $2x + (m^2 - 1)y - z - 1 = 0$ where m is a real parameter

1. Show that, for any m, plane (P_m) passes through the fixed line of equation $2x - z - 1 = 0$, lying in the xz-plane.
2. Determine the values of m so that the line of intersection of (P_m) with the xy – plane is $2x + 3y - 1 = 0$
2. Let (R) be a plane of equation $2x - 4y - z = 0$
   a) Determine m so that (P_m) is parallel to (R).
   b) Determine m so that (P_m) is perpendicular to (R).
4. Consider the plane (P_1) which corresponds to to m = 1 in the set of planes (P_m) and the plane (Q) of equation $x + y - 3 = 0$.
   a) Verify that both points A(2; 1; 3) and B(3; 0; 5) belong to planes (P_1) and (Q).
   b) Find the parametric equations of (L), the line of intersection of planes (P_1) and (Q).
   c) Does plane (Q) intersect the z – axis? Verify.
   d) Verify that the line

   (D): $\begin{cases} x = t + 1 \\ y = 1 \\ z = 2t + 1 \end{cases}$ belongs to plane (P_1)

   e) Does point A(2; 1; 3) belong to line (D)? Verify.
   Deduce the point of intersection of lines (D) and (L).

V-(6 points). Box A contains 9 items of which 3 are defective, and box B contains 7 items of which 3 are defective.

1. Two items are drawn at random from box A. Consider the following events:
   DD: the 2 items are defective
   NN: the 2 items are non-defective
   DN: one item is defective and one not
   a) Calculate P(DD), the probability of event DD
   b) Calculate P(NN), the probability of event NN
   c) Calculate P(DN), the probability of event DN
2. In this part, we draw randomly 3 items from box B as follows:
   We draw the first item without putting it back in the box, then we draw the second item and we put it back in the box and finally, we draw the third item.
   Verify that the probability of drawing 3 defective items is $1/21$
3. An item is drawn at random from each box.
   a) What is the probability P_1 that both items are defective?
   b) What is the probability P_2 that one item defective and one not?